ERRATA FOR 'THE THEORY OF NILPOTENT GROUPS'

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- p. 12. In Example 1.9: Instead of "Recall from Example 1.6" it should read "Recall from Example 1.4".
- p. 39. The statements of Lemmas 2.10 and 2.11 should contain equalities, not isomorphisms. Some of the isomorphisms in their proofs should also be equalities.
- p. 48. In Theorem 2.12-(*ii*), replace "$|H| = p^n$" with "$|H| = p^i$ for some $i \leq n$".
- p. 48. At the beginning of the proof of Theorem 2.13, one must replace "$N_G(P) < M$" with "$N_G(P) \leq M$", to allow for the possibility that $N_G(P)$ be maximal.
- p. 49. The proof that (*vii*) implies (*vi*) is incorrect. The issue is the claim that "$|g|$ is coprime to $|h|$". We cannot assume that P_i is the unique Sylow subgroup associated to p_i. To fix this, we prove that (*vii*) implies (*v*), from which it follows that (*vii*) implies (*vi*).

  Suppose that the elements of G of coprime order commute. Let
  $$|G| = p_1^{n_1} p_2^{n_2} \cdots p_r^{n_r},$$
  where $p_1, p_2, \ldots, p_r$ are distinct primes, and let $1 \neq g \in G$ with
  $$|g| = p_1^{m_1} p_2^{m_2} \cdots p_r^{m_r}$$
  for some $0 \leq m_i \leq n_i$. Set $q_i = \dfrac{|g|}{p_i^{m_i}}$. Note that $q_i = |g|$ if $m_i = 0$, but at least one of the m_j is not zero. Clearly, $\gcd(q_1, q_2, \ldots, q_r) = 1$. Thus, there are integers $a_1, a_2, \ldots, a_r$ such that $a_1 q_1 + a_2 q_2 + \cdots + a_r q_r = 1$. Hence,
  $$g = g^1 = g^{a_1 q_1 + a_2 q_2 + \cdots + a_r q_r} = g^{a_1 q_1} g^{a_2 q_2} \cdots g^{a_r q_r}.$$
  Put $g_i = g^{a_i q_i}$. Then
  $$g = g_1 g_2 \cdots g_r \quad \text{and} \quad g_i^{p_i^{m_i}} = g^{a_i q_i p_i^{m_i}} = g^{a_i |g|} = 1.$$
  Thus, $|g_i|$ divides $p_i^{m_i}$. And so, by Theorem 2.12-(*ii*), there is a Sylow p_i-subgroup P_i containing g_i. Hence, $g \in P_1 P_2 \cdots P_r$. Now, P_i and P_j have coprime order whenever $i \neq j$. So, by hypothesis, the elements of P_i commute with those of P_j.

  We claim that $P_i \trianglelefteq G$. Let $x_i \in P_i$. Since $g = g_1 \ldots g_r$, we have
  $$g^{-1} x_i g = g_r^{-1} \ldots g_1^{-1} x_i g_1 \ldots g_r = g_r^{-1} \ldots g_{i+1}^{-1} \overline{x_i} g_{i+1} \ldots g_r = \overline{x_i},$$
  where $\overline{x_i} = g_i^{-1} x_i g_i \in P_i$. So P_i is indeed normal in G. It now follows from Corollary 2.8 that P_i is the only Sylow p_i-subgroup of G. We conclude that all Sylow subgroups of G are normal.

- p. 282. (7.9) is incorrect. Replace said line with

$$\phi(xG_i) = xG_i \text{ for every } i = 1, \ldots, r \text{ and } x \in G_{i-1}.$$

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